Heat Transfer

Unit I I

Introduction and Heat Conduction

Internal heat generation is the one, where heat is uniformly generated throughout the material at a constant rate (expressed as W/m^3).

Examples : 1. Heat generated due to passage of current through the metals like electrical conductor.

2. Heat generated due to fission or fusion reaction in Nuclear fuel.

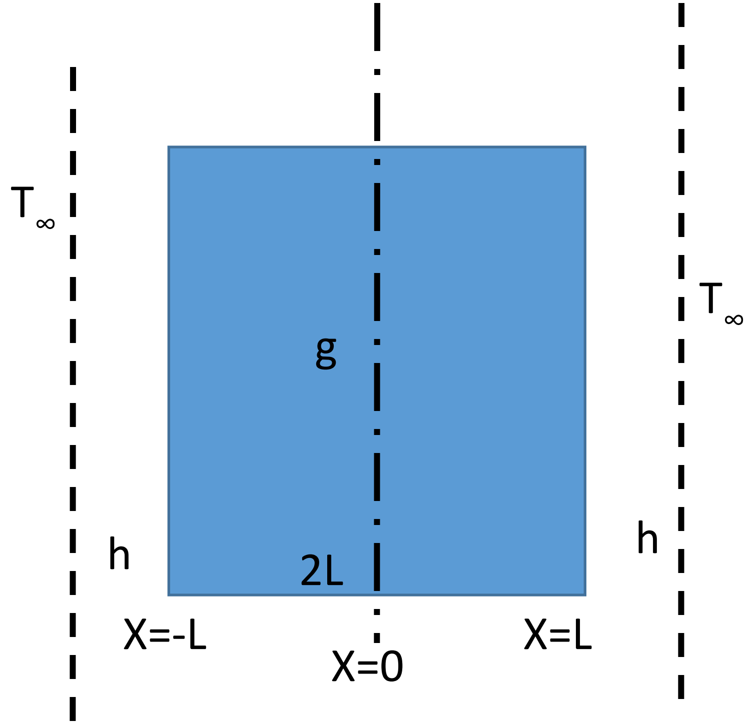
3. Setting of concrete slab by releasing heat uniformly.

4. Combustion of fuel in IC Engines

Heat Conduction with Internal Heat Generation Through A Slab (Symmetrical BCs)

Consider an infinite slab of thickness 2L. Let g (W/m^3) be internal heat generation at const rate and same surrounding fluid both sides be at temp T_∞ .

For convenience, x=0 has been aligned with centre line of thickness of slab; so that left face of the slab is at x=-L and right face at x=L



Poisson's Eqn applicable in the present case is:

$$\frac{d^2T}{dx^2} + \frac{g}{k} = 0$$

Aim is to find out Temp Distr $T_{(x)}$ through Slab and Heat Flow Rate Q

Applicable equation $$\frac{d^2T}{dx^2} + \frac{g}{k} = 0$$

OR $$\frac{d^2T}{dx^2} = -\frac{g}{k}$$

Integrating twice, We have;

$$\frac{dT}{dx} = -\frac{g}{k}.x + C_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

And $$T = -\frac{g}{2k}.x^2 + C_1.x + C_2 \ldots\ldots(2)$$

Boundary Conditions:

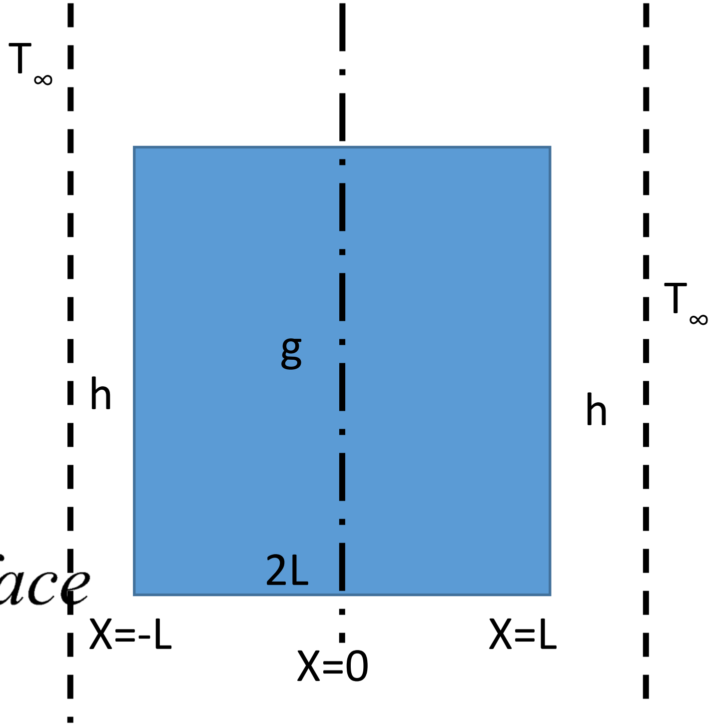
$$1.\quad \frac{dT}{dx} = 0 \quad at\ x = 0;\ as\ Temp$$

$$will\ be\ \max at\ the\ centre\ as\ same$$

$$conditions\ exist\ on\ both\ sides\ of\ slab$$

$$2.\ Heat\ Conducted\ upto\ face = Heat\ Convected\ from\ the\ face$$

$$\left[-kA\frac{dT}{dx}\right]_{x=L} = hA.\left[T_L - T_\infty\right]_{x=L}$$



$$From\ Eqn...(1), \frac{dT}{dx} = 0\ for\ x = 0 \Rightarrow C_1 = 0$$

$$Hence Eqn...(2)\ becomes\ T = -\frac{g}{2k}.x^2 + C_2$$

$$Applying\ \ BC...(2), We\ have;$$

$$-kA\left(\frac{-gL}{k}\right) = hA\left(\frac{-gL^2}{2k} + C_2 - T_\infty\right)$$

$$\Rightarrow C_2 = \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty$$

$$Substituting\ C_2 = \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty in\ \text{modified eqn...(2)}$$

$$We\ have: T = -\frac{gx^2}{2k} + \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty$$

$$T = \frac{g}{2k}\left(L^2 - x^2\right) + \frac{gL}{h} + T_\infty \text{...Temp Distribution}$$

Max Temp will occur at Centre (At $x = 0$);

$$\text{Hence} \quad T_{max} = \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty$$

For Surface Temp, putting $x = L$,

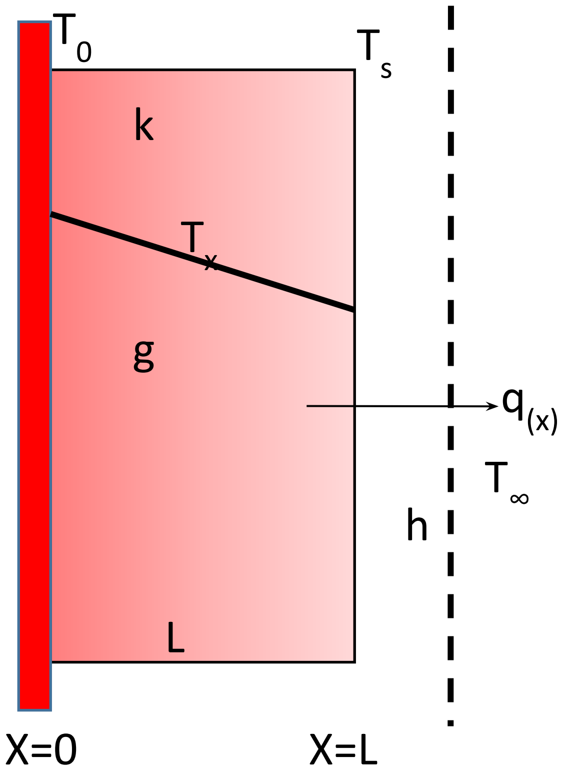
$$\text{We have} \quad T_s = \frac{gL}{h} + T_\infty$$

Heat Flux

$q_{(x)} = g.x$

Q5:Consider a slab of thickness L and k conductivity, in which energy is generated at a const rate of g W/m^3 . Surface at x=0 is insulated and surface at x=L dissipates heat by convection with h to a fluid at T_∞

We have to find out Temp Distr $T_{(x)}$ through slab & heat flux $q_{(x)}$



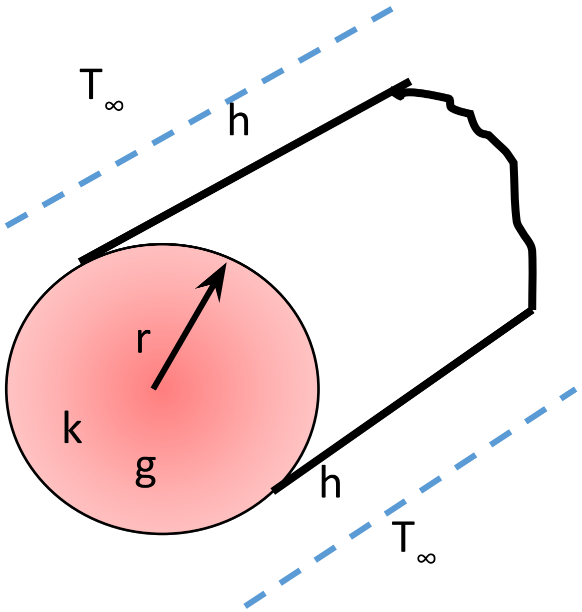
Also, We have to calculate temp T_0 at x=0 and T_s at x=L for L=1cm, k=20 W/mK, g=$8x10^7$ W/m^3, h=4000 W/m^2K & T_∞=100°C

Heat Conduction with Internal Heat Generation Through A Long Solid Cylinder (Symmetrical BCs)

Consider a solid cylinder of radius r of conductivity k, in which internal heat is generated at a const rate of g W/m^3 .

This cylinder is exposed to a fluid with heat transfer coefficient h at temp T_∞ , to which it is dissipating heat by convection

Example: An electric conductor carrying current exposed to atmospheric air



$$\text{Poisson's Equation}: \frac{1}{r}.\frac{d}{dr}\left(r.\frac{dT}{dr}\right)+\frac{g}{k}=0$$

$$\text{or} \quad \frac{d}{dr}\left(r.\frac{dT}{dr}\right)=-\frac{gr}{k}$$

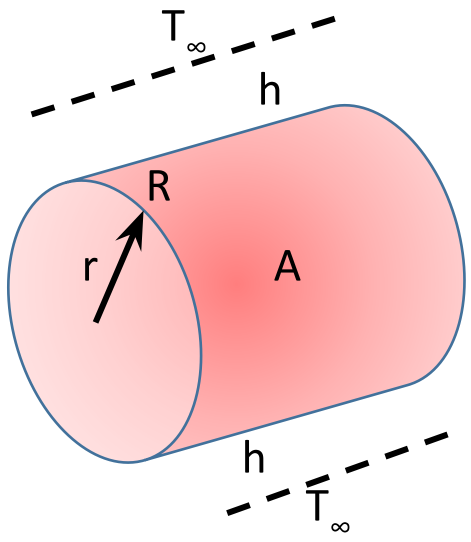
Integrating above Eqn Twice;

$$r.\frac{dT}{dr}=-\frac{gr^2}{2k}+C_1 \quad \text{or}$$

$$\frac{dT}{dr}=-\frac{gr}{2k}+\frac{C_1}{r}\;..........(1) \quad \text{and}$$

$$T=\frac{-gr^2}{4k}+C_1 \ln r+C_2\;.........(2)$$

Boundary Conditions



$$1.\ \frac{dT}{dr} = 0 \text{ at } r = 0; \text{ because Temp will be}$$

max at the centre as same conditions

exist on all sides of cylinder $\Rightarrow C_1 = 0$

2. Heat Conducted upto surface of the cylinder

= Heat Convected from the surface of the cylinder

$$\left[-kA\frac{dT}{dr}\right]_{r=R} = hA.\left[T_R - T_\infty\right]_{r=R}$$

$$Substituting\ \frac{dT}{dr}\ \&\ T_r$$

$$at\ r = R\ from\ eqns.(1)\ \&\ (2), we\ have \Rightarrow C_2 = \frac{gR^2}{4k} + \frac{gR}{2h} + T_\infty$$

$$Substituting\ C_1\ \&\ C_2\ in\ Equation..(2);$$

$$We\ have: T = \frac{g}{4k}\left(R^2 - r^2\right) + \frac{gR}{2h} + T_\infty$$

$$Max\ Temp\ (at\ Centre) \quad T_{\max(r=0)} = \frac{gR^2}{4k} + \frac{gR}{2h} + T_\infty$$

$$Temp\ at\ surface\ (r = R) \quad T_s = \frac{gR}{2h} + T_\infty$$

$$Heat\ Flux \quad q_{(r)} = \left[-k\frac{dT}{dr}\right]_{r=R}$$

$$= -k.\left(\frac{-gR}{2k}\right) = \frac{g.R}{2} \quad W/m^2$$

$$Q = -k.A.\left[\frac{dT}{dr}\right]_{r=R} = -k.2\pi R\left(\frac{-gR}{2k}\right)$$

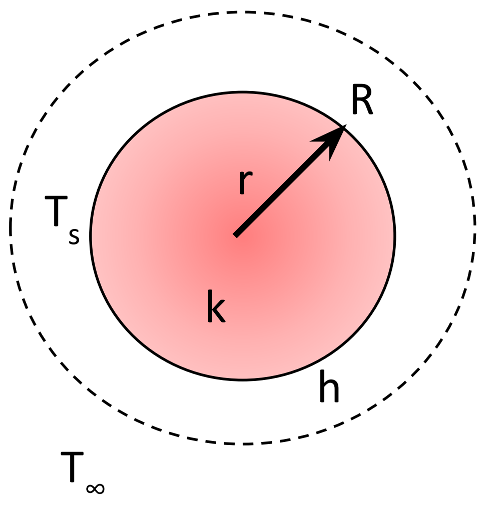
$$= g.\pi R^2(1m) = g.Volume.....W/m$$

Heat Conduction with Internal Heat Generation Through A Solid Sphere (Symmetrical BCs)

Poison’s Equation for sphere:

$$\frac{1}{r^2}\cdot\frac{d}{dr}\cdot\left(r^2\frac{dT}{dr}\right)+\frac{g}{k}=0$$

$$\text{OR}\quad \frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{gr^2}{k}$$



On Integrating twice; $r^2\frac{dT}{dr}=-\frac{gr^3}{3k}+C_1$

$$or\quad \frac{dT}{dr}=-\frac{gr}{3k}+\frac{C_1}{r^2}.........(1)$$

$$And\quad T=-\frac{gr^2}{6k}-\frac{C_1}{r}+C_2..........(2)$$

Boundary Conditions:

$$1. \quad \frac{dT}{dr} = 0 \; at \quad r = 0;$$

$$\Rightarrow C_1 = 0 \; from \; Eqn...(1)$$

$$2. \quad Heat \; Conducted \; upto \; Surface$$

$$= Heat \; Convected \; out \; from \; the \; Surface$$

$$Hence \quad -k.A.\left[\frac{dT}{dr}\right]_{r=R} = h.A.\left[T_s - T_\infty\right]_{r=R}$$

$$\Rightarrow C_2 = \frac{gR^2}{6k} + \frac{gR}{3h} + T_\infty$$

Substituting values of C_1 & C_2 *in Eqn...(2)*

$$We\ have; \quad T = -\frac{gr^2}{6k} + \frac{gR^2}{6k} + \frac{gR}{3h} + T_\infty$$

$$or \qquad T = \frac{g}{6k}\left(R^2 - r^2\right) + \frac{gR}{3h} + T_\infty$$

$$q_{(r)} = -k.\left(\frac{dT}{dr}\right) = -k.\left(\frac{-gr}{3k}\right) = \frac{gr}{3} \quad W/m^2$$

Heat Conduction with Internal Heat Generation

Slab:

$$T = \frac{g}{2k}.\left(L^2 - x^2\right) + \frac{gL}{h} + T_\infty; \quad q_{(x)} = g.x \quad W/m^2$$

Cylinder:

$$T = \frac{g}{4k}\left(R^2 - r^2\right) + \frac{gR}{2h} + T_\infty; \; q_{(r)} = \frac{gr}{2} \quad W/m^2$$

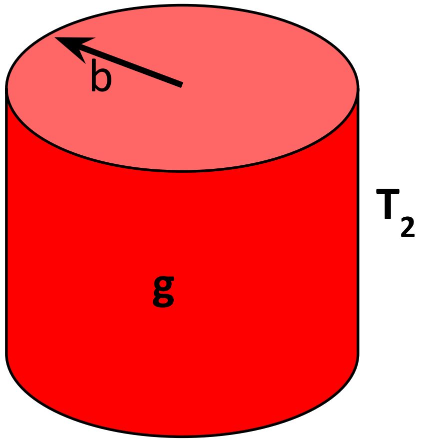
Sphere:

$$T = \frac{g}{6k}\left(R^2 - r^2\right) + \frac{gR}{3h} + T_\infty; \quad q_{(r)} = \frac{gr}{3} \quad W/m^2$$

Q6: Consider a solid cylinder of radius $r=b$, in which energy is generated at a const rate of g W/m^3 while boundary surface at $r=b$ is maintained at temp T_2.

Develop an expression for one dimensional(radial), steady state temp distr $T_{(r)}$ and heat flux $q_{(r)}$.

Calculate centre temp and heat flux at the boundary surface $r=b$ for $b=1cm$, $g=2x10^8$ W/m^3, $k=20W/mK$, & $T_2=100°C$



Q7: Heat is generated in a solid sphere of 10 cm dia. at the rate of 600 W/m^3. Surface heat transfer coeff. is 10 W/m^2K and surrounding air temp 30°C. k of material is 0.2W/mK .

Find:

a) Max temp in the sphere
b) Surface temp of sphere